

# There Are Many Ways to Represent Functions

## Recognizing Algebraic and Graphical Representations of Functions

### LEARNING GOALS

In this lesson, you will:

- Write equations using function notation.
- Recognize multiple representations of functions.
- Determine and recognize characteristics of functions.
- Determine and recognize characteristics of function families.

### KEY TERMS

- function notation
- increasing function
- decreasing function
- constant function
- function family
- linear functions
- exponential functions
- absolute minimum
- absolute maximum
- quadratic functions
- linear absolute value functions
- linear piecewise functions

Just about everything you see, hear, or own has a name. It's not just people who have names—streets have names, cars have names, even trees have names. So where do these names come from and why were they chosen? There are many naming conventions we use in our society. The purpose of these naming conventions is to provide useful information about the object being named. For example, just saying “I live on a street” does not provide much information. However, saying “I live on East Main Street” makes it much more clear where you live.

Think about other objects and their names. Why do you think they were named the way they were? What information is provided by their names? Would another name suit the object better?

## PROBLEM 1 A New Way to Write Something Familiar



Functions can be represented in a number of ways. An equation representing a function can be written using *function notation*. **Function notation** is a way of representing functions algebraically. This form allows you to more efficiently identify the independent and dependent quantities. The function  $f(x)$  is read as “ $f$  of  $x$ ” and indicates that  $x$  is the independent variable.

Let’s look at the relationship between an equation and function notation.

Consider orders for a custom T-shirt shop. U.S. Shirts charges \$8 per shirt plus a one-time charge of \$15 to set a T-shirt design. The equation  $y = 8x + 15$  can be written to model this situation. The independent variable  $x$  represents the number of shirts ordered, and the dependent variable  $y$  represents the total cost of the order, in dollars.

You know this is a function because for each number of shirts ordered (independent value) there is exactly one total cost (dependent value) associated with it.

Because this situation is a function, you can write  $y = 8x + 15$  in function notation.

$$f(x) = 8x + 15$$

The cost, defined by  $f$ , is a function of  $x$ , the number of shirts ordered.

So can I use any two letters when representing a function?


A common way to name a function is  $f(x)$ . However, you can choose any variable to name a function. You could write the T-shirt cost function as  $C(s) = 8s + 15$ , where the cost, defined as  $C$ , is a function of  $s$ , the number of shirts ordered.

Remember, you can only write *functions* in function notation. So sorry, *non-functions*! You’ll still need to be written as equations.



You can input equations written in function notation into your graphing calculator. Your graphing calculator will list different functions as  $Y_1, Y_2, Y_3,$  etc.

Let's graph the function  $f(x) = 8x + 15$  on a calculator by following the steps shown.

 You can use a graphing calculator to graph a function.

**Step 1:** Press **Y=**. Your cursor should be blinking on the line **\Y1=**. Enter the equation. To enter a variable like  $x$ , press the key with **X**, **T**, **0**, **n** once.

**Step 2:** Press **WINDOW** to set the bounds and intervals you want displayed.

**Step 3:** Press **GRAPH** to view the graph.

The way you set the window will vary each time depending on the equation you are graphing.

```

WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-20
Ymax=20
Yscl=2
    
```

The **Xmin** represents the least point on the  $x$ -axis that will be seen on the screen. The **Xmax** represents the greatest point that will be seen on the  $x$ -axis. Lastly, the **Xscl** represents the intervals. Similar names are used for the  $y$ -axis (**Ymin**, **Ymax**, and **Yscl**).

A convention to communicate the viewing **WINDOW** on a graphing calculator is shown.

$$\begin{array}{l}
 \text{Xmin: } -10 \\
 \text{Xmax: } 10
 \end{array}
 \left. \vphantom{\begin{array}{l} \text{Xmin: } -10 \\ \text{Xmax: } 10 \end{array}} \right\} [-10, 10]
 \left. \vphantom{\begin{array}{l} [-10, 10] \\ [-20, 20] \end{array}} \right\} [-10, 10] \times [-20, 20]$$

$$\begin{array}{l}
 \text{Ymin: } -20 \\
 \text{Ymax: } 20
 \end{array}
 \left. \vphantom{\begin{array}{l} \text{Ymin: } -20 \\ \text{Ymax: } 20 \end{array}} \right\} [-20, 20]$$

## PROBLEM 2 Up, Down, or Neither?



In the previous lesson, you determined which of the given graphs represented functions. Gather all of the graphs from the previous lesson that you identified as functions.

A function is described as increasing when the dependent variable increases as the independent variable increases. If a function increases across the entire domain, then the function is called an **increasing function**.

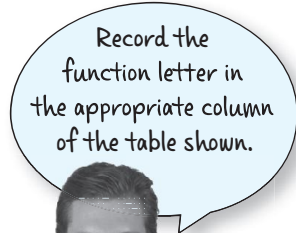
A function is described as decreasing when the dependent variable decreases as the independent variable increases. If a function decreases across the entire domain, then the function is called a **decreasing function**.

If the dependent variable of a function does not change or remains constant over the entire domain, then the function is called a **constant function**.



1. Analyze each graph from left to right. Sort all the graphs into one of the four groups:

- increasing function,
- decreasing function,
- constant function,
- a combination of increasing, decreasing, or constant.



Increasing Function	Decreasing Function	Constant Function	Combination of Increasing, Decreasing, or Constant

2. Each function shown represents one of the graphs in the increasing function, decreasing function, or constant function categories. Enter each function into a graphing calculator to determine the shape of its graph. Then match the function to its corresponding graph by writing the function directly on the graph that it represents.

- $f(x) = x$
- $f(x) = \left(\frac{1}{2}\right)^x - 5$
- $f(x) = 2^x$ , where  $x$  is an integer
- $f(x) = -\frac{2}{3}x + 5$
- $f(x) = -x + 3$ , where  $x$  is an integer
- $f(x) = \left(\frac{1}{2}\right)^x$
- $f(x) = 2$ , where  $x$  is an integer

Be sure to correctly interpret the domain of each function. Also, remember to use parentheses when entering fractions.



3. Consider the seven graphs and functions that are increasing functions, decreasing functions, or constant functions.

a. Sort the graphs into two groups based on the equations representing the functions and record the function letter in the table.

Group 1	Group 2



b. What is the same about all the functions in each group?



Congratulations! You have just sorted the graphs into their own *function families*. A **function family** is a group of functions that share certain characteristics.

The family of **linear functions** includes functions of the form  $f(x) = mx + b$ , where  $m$  and  $b$  are real numbers.

The family of **exponential functions** includes functions of the form  $f(x) = a \cdot b^x$ , where  $a$  and  $b$  are real numbers, and  $b$  is greater than 0 but is not equal to 1.

4. Go back to your table in Question 3 and identify which group represents linear and constant functions and which group represents exponential functions.
  
5. If  $f(x) = mx + b$ , represents a linear function, describe the  $m$  and  $b$  values that produce a constant function.

Place these two groups of graphs off to the side. You will need them again in Problem 4. Wait for it...



### PROBLEM 3 Least, Greatest, or Neither?



A function has an **absolute minimum** if there is a point that has a  $y$ -coordinate that is less than the  $y$ -coordinates of every other point on the graph. A function has an **absolute maximum** if there is a point that has a  $y$ -coordinate that is greater than the  $y$ -coordinates of every other point on the graph.



1. Sort the graphs from the Combination category in Problem 2 into three groups:
  - those that have an absolute minimum value,
  - those that have an absolute maximum value, and
  - those that have no absolute minimum or maximum value.

Then record the function letter in the appropriate column of the table shown.

Think about the graphical behavior of the function over its entire domain.

Absolute Minimum	Absolute Maximum	No Absolute Minimum or Absolute Maximum



2. Each function shown represents one of the graphs with an absolute maximum or an absolute minimum value. Enter each function into your graphing calculator to determine the shape of its graph. Then match the function to its corresponding graph by writing the function directly on the graph that it represents.

- $f(x) = x^2 + 8x + 12$
- $f(x) = |x - 3| - 2$
- $f(x) = x^2$
- $f(x) = |x|$
- $f(x) = -|x|$
- $f(x) = -3x^2 + 4$ , where  $x$  is integer
- $f(x) = -\frac{1}{2}x^2 + 2x$
- $f(x) = -2|x + 2| + 4$

When entering an absolute value function into your calculator, Press **2nd Catalog, abs(**. Then type the absolute value portion of the equation and press **)**. For example, the second function would look like this:  
 $Y_1 = \text{abs}(x - 3) - 2$



3. Consider the graphs of functions that have an absolute minimum or an absolute maximum. (Do not consider Graphs A and C yet.)

a. Sort the graphs into two groups based on the equations representing the functions and record the function letter in the table.

Group 1	Group 2



b. What is the same about all the functions in each group?



Congratulations! You have just sorted functions into two more function families.

The family of **quadratic functions** includes functions of the form  $f(x) = ax^2 + bx + c$ , where  $a$ ,  $b$ , and  $c$  are real numbers, and  $a$  is not equal to 0.

The family of **linear absolute value functions** includes functions of the form  $f(x) = a|x + b| + c$ , where  $a$ ,  $b$ , and  $c$  are real numbers, and  $a$  is not equal to 0.

- Go back to your table in Question 3 and identify which group represents quadratic functions and which group represents linear absolute value functions.

## PROBLEM 4 Piecing Things Together



Analyze each of the functions shown. These functions represent the last three graphs of functions from the no absolute minimum and no absolute maximum category.

$$\bullet f(x) = \begin{cases} -2x + 10, & -\infty \leq x < 3 \\ 4, & 3 \leq x < 7 \\ -2x + 18, & 7 \leq x \leq +\infty \end{cases}$$

$$\bullet f(x) = \begin{cases} -2, & -\infty < x < 0 \\ \frac{1}{2}x - 2, & 0 \leq x < \infty \end{cases}$$

$$\bullet f(x) = \begin{cases} \frac{1}{2}x + 4, & -\infty \leq x < 2 \\ -3x + 11, & 2 \leq x < 3 \\ \frac{1}{2}x + \frac{1}{2}, & 3 \leq x \leq \infty \end{cases}$$

If your graphing calculator does not have an infinity symbol, you can enter the biggest number your calculator can compute using scientific notation. On mine, this is  $9 \times 10^{99}$ . I enter this by pressing **9 2nd EE 99**, which is shown on my calculator as **9E99**.



These functions are part of the family of *linear piecewise functions*. **Linear piecewise functions** include functions that have equation changes for different parts, or pieces, of the domain.

Because these graphs each contain compound inequalities, there are additional steps required to use a graphing calculator to graph each function.

So then for negative infinity, would I use -9 times 10 to the 99th power? How do I enter that?





Let's graph the piecewise function:

$$f(x) = \begin{cases} -2x + 10, & -\infty \leq x < 3 \\ 4, & 3 \leq x < 7 \\ -2x + 18, & 7 \leq x \leq +\infty \end{cases}$$



You can use a graphing calculator to graph piecewise functions.

The first section is  $(-2x + 10)$ .

The first part of the inequality is  $(-\infty \leq x)$ .

**Step 1:** Press **Y=**. Enter the first section of the function within parentheses. Then press the division button.

**Step 2:** Press the **(** key twice and enter the first part of the compound inequality within parentheses.

**Step 3:** Enter the second part of the compound inequality within parentheses and then type two closing parentheses.

Press **GRAPH** here to see the first section of the piecewise function.

**Step 4:** Enter the remaining sections of the piecewise functions as  $Y_2$  and  $Y_3$ .

Use the **2ND TEST** to locate the inequality symbols. The **TEST** button is on the same key as the **MATH** button.

The second part of the compound inequality is  $(x < 3)$ .  
The entire line should look like:  
 $Y_1 = (-2x + 10) / ((-\infty \leq x) (x < 3))$

By completing the first piecewise function, you can now choose the graph that matches your graphing calculator screen.



1. Enter the remaining functions into your graphing calculator to determine the shapes of their graphs.
2. Match each function to its corresponding graph by writing the function directly on the graph that it represents.

You will need these graphs again in Problem 5. Wait for it. . .



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Congratulations! You have just sorted the remaining functions into one more function family.

The family of linear piecewise functions includes functions that have equation changes for different parts, or pieces, of the domain.

You will need these graphs again in Problem 5. Wait for it...



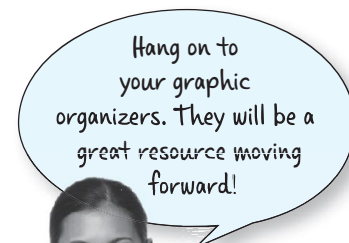
## PROBLEM 5 We Are Family!

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You have now sorted each of the graphs and equations representing functions into one of five function families: linear, exponential, quadratic, linear absolute value, and linear piecewise.



1. Glue your sorted graphs and functions to the appropriate function family Graphic Organizer on the pages that follow. Write a description of the graphical behavior for each function family.



Hang on to your graphic organizers. They will be a great resource moving forward!

You've done a lot of work up to this point! You've been introduced to linear, exponential, quadratic, linear absolute value, and linear piecewise functions. Don't worry—you don't need to know everything there is to know about all of the function families right now. As you progress through this course, you will learn more about each function family.



Be prepared to share your solutions and methods.



### Definition

The family of **linear functions** includes functions of the form  $f(x) = mx + b$ , where  $m$  and  $b$  are real numbers.

### Graphical Behavior

**Increasing / Decreasing:**

**Maximum / Minimum:**

**Curve / Line:**

## Linear Functions

Examples

### Definition

The family of **exponential functions** includes functions of the form  $f(x) = a \cdot b^x$ , where  $a$  and  $b$  are real numbers, and  $b$  is greater than 0 but not equal to 1.

### Graphical Behavior

**Increasing / Decreasing:**

**Maximum / Minimum:**

**Curve / Line:**

## Exponential Functions

### Examples

### Definition

The family of **quadratic functions** includes functions of the form,  $f(x) = ax^2 + bx + c$  where  $a$ ,  $b$ , and  $c$  are real numbers, and  $a$  is not equal to 0.

### Graphical Behavior

**Increasing / Decreasing:**

**Maximum / Minimum:**

**Curve / Line:**

## Quadratic Functions

### Examples

### Definition

The family of **linear absolute value functions** includes functions of the form  $f(x) = a|x + b| + c$ , where  $a$ ,  $b$ , and  $c$  are real numbers, and  $a$  is not equal to 0.

### Graphical Behavior

**Increasing / Decreasing:**

**Maximum / Minimum:**

**Curve / Line:**

## Linear Absolute Value Functions

### Examples



### Definition

The family of **linear piecewise functions** includes functions that have equation changes for different parts, or pieces, of the domain.

### Graphical Behavior

**Increasing / Decreasing:**

**Maximum / Minimum:**

**Curve / Line:**

## Linear Piecewise Functions

Examples

