The Plane!

Modeling Linear Situations

2.1

LEARNING GOALS

In this lesson, you will:

- Complete tables and graphs, and write equations to model linear situations.
- Analyze multiple representations of linear relationships.
- Identify units of measure associated with linear relationships.
- Determine solutions both graphically and algebraically.
- Determine solutions to linear functions using intersection points.

KEY TERMS

- first differences
- solution
- intersection point

"Ladies and gentlemen, at this time we ask that all cell phones and pagers be turned off for the duration of the flight. All other electronic devices must be turned off until the aircraft reaches 10,000 feet. We will notify you when it is safe to use such devices."

Flight attendants routinely make announcements like this on airplanes shortly before takeoff and landing. But what's so special about 10,000 feet?

When a commercial airplane is at or below 10,000 feet, it is commonly known as a "critical phase" of flight. This is because research has shown that most accidents happen during this phase of the flight—either takeoff or landing. During critical phases of flight, the pilots and crew members are not allowed to perform any duties that are not absolutely essential to operating the airplane safely.

And it is still not known how much interference cell phones cause to a plane's instruments. So, to play it safe, crews will ask you to turn them off.

PROBLEM 1 Analyzing Tables

A 747 airliner has an initial climb rate of 1800 feet per minute until it reaches a height of 10,000 feet.

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1. Identify the independent and dependent quantities in this problem situation. Explain your reasoning.

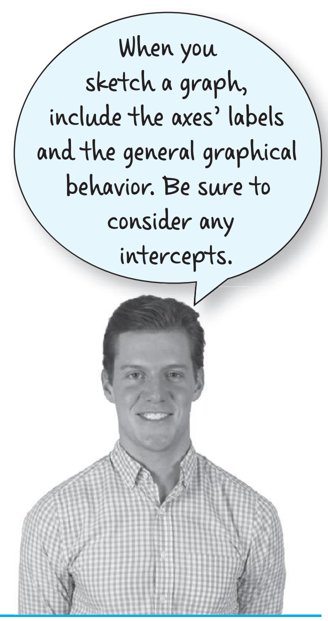
2. Describe the units of measure for:

   a. the independent quantity (the input values).

   b. the dependent quantity (the output values).

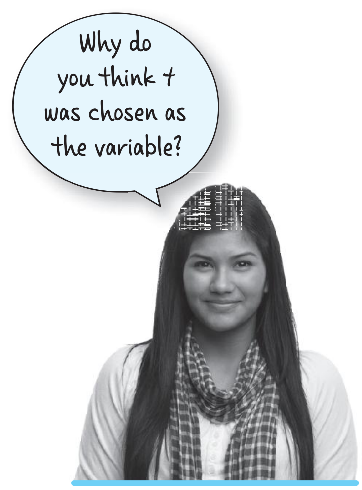
3. Which function family do you think best represents this situation? Explain your reasoning.

4. Draw and label two axes with the independent and dependent quantities and their units of measure. Then sketch a simple graph of the function represented by the situation.



5. Write the independent and dependent quantities and their units of measure in the table. Then, calculate the dependent quantity values for each of the independent quantity values given.

	Independent Quantity	Dependent Quantity
Quantity		
Units		
	0	
	1	
	2	
	2.5	
	3	
	3.5	
	5	
Expression	t	



6. Write an expression in the last row of the table to represent the dependent quantity. Explain how you determined the expression.

Let's examine the table to determine the unit rate of change for this situation. One way to determine the unit rate of change is to calculate *first differences*. Recall that **first differences** are determined by calculating the difference between successive points.

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7. Determine the first differences in the section of the table shown.

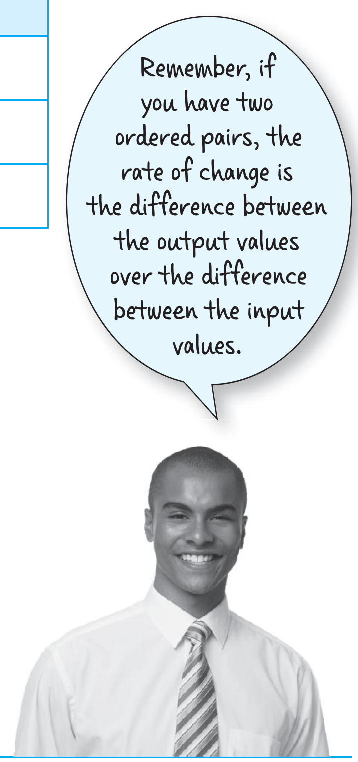
	Time (minutes)	Height (feet)	First Differences
	0	0	
1 − 0 = 1			
	1	1800	
2 − 1 = 1			
	2	3600	
3 − 2 = 1			
	3	5400	

8. What do you notice about the first differences in the table? Explain what this means.

Another way to determine the unit rate of change is to calculate the rate of change between any two ordered pairs and then write each rate with a denominator of 1.

9. Calculate the rate of change between the points represented by the given ordered pairs in the section of the table shown. Show your work.

Time (minutes)	Height (feet)
2.5	4500
3	5400
5	9000



a. (2.5, 4500) and (3, 5400)

b. (3, 5400) and (5, 9000)

c. (2.5, 4500) and (5, 9000)

10. What do you notice about the rates of change?

11. Use your answers from Question 7 through Question 10 to describe the difference between a rate of change and a unit rate of change.

12. How do the first differences and the rates of change between ordered pairs demonstrate that the situation represents a linear function? Explain your reasoning.

13. Alita says that in order for a car to keep up with the plane on the ground, it would have to travel at only 20.5 miles per hour. Is Alita correct? Why or why not?

PROBLEM 2 Analyzing Equations and Graphs

1. Complete the table shown for the problem situation described in Problem 1, *Analyzing Tables*. First, determine the unit of measure for each expression. Then, describe the contextual meaning of each part of the function. Finally, choose a term from the word box to describe the mathematical meaning of each part of the function.

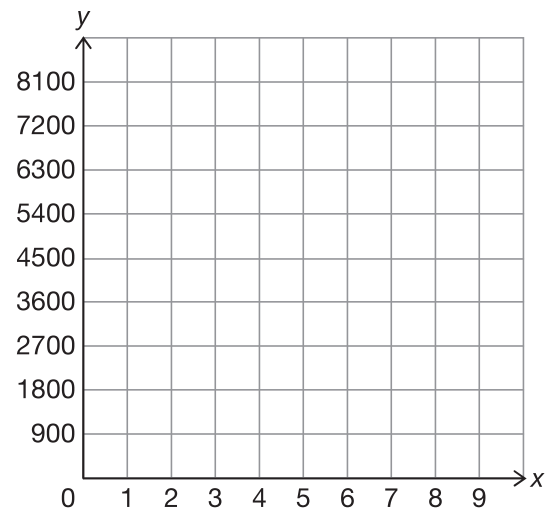
output value	input value	rate of change

		What It Means	
Expression	**Unit**	**Contextual Meaning**	**Mathematical Meaning**
t			
1800			
$1800t$			

2. Write a function, $h(t)$, to describe the plane's height over time, t.

3. Which function family does $h(t)$ belong to? Is this what you predicted back in Problem 1, Question 3?

4. Use your table and function to create a graph to represent the change in the plane's height as a function of time. Be sure to label your axes with the correct units of measure and write the function.



a. What is the slope of this graph? Explain how you know.

b. What is the x-intercept of this graph? What is the y-intercept? Explain how you determined each intercept.

c. What do the x- and y-intercepts mean in terms of this problem situation?

Let's consider how to determine the height of the plane, given a time in minutes, using function notation.

To determine the height of the plane at 2 minutes using your function, substitute 2 for t every time you see it. Then, simplify the function.

$$h(t) = 1800t$$

Substitute 2 for t. ⟶ $h(2) = 1800(2)$

$$h(2) = 3600$$

Two minutes after takeoff, the plane is at 3600 feet.

5. List the different ways the height of the plane is represented in the example.

6. Use your function to determine the height of the plane at each given time in minutes. Write a complete sentence to interpret your solution in terms of the problem situation.

a. $h(3) =$ ____________ **b.** $h(3.75) =$ ____________

c. $h(5.1) =$ ____________ **d.** $h(-4) =$ ____________

PROBLEM 3 Connecting Approaches

Now let's consider how to determine the number of minutes the plane has been flying (the input value) given a height in feet (the output value) using function notation.

To determine the number of minutes it takes the plane to reach 7200 feet using your function, substitute 7200 for $h(t)$ and solve.

$$h(t) = 1800t$$

Substitute 7200 for $h(t)$. →

$$7200 = 1800t$$

$$\frac{7200}{1800} = \frac{1800t}{1800}$$

$$4 = t$$

After takeoff, it takes the plane 4 minutes to reach a height of 7200 feet.

1. Why can you substitute 7200 for $h(t)$?

2. Use your function to determine the time it will take the plane to reach each given height in feet. Write a complete sentence to interpret your solution in terms of the problem situation.

   a. 5400 feet

   b. 9000 feet

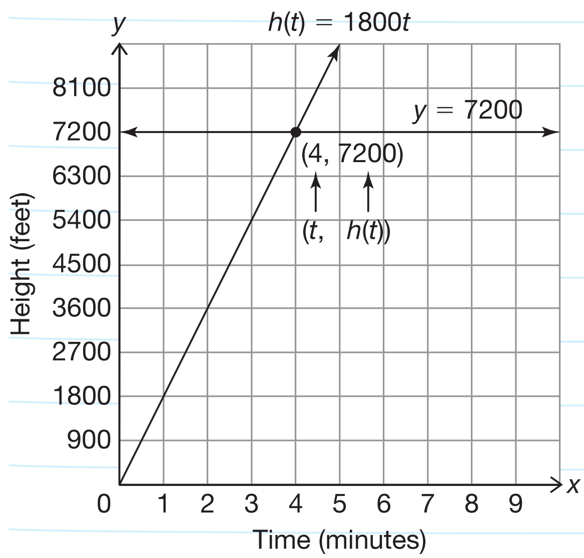
   c. 3150 feet

   d. 4500 feet

You can also use the graph to determine the number of minutes the plane has been flying (input value) given a height in feet (output value). Remember, the **solution** of a linear equation is any value that makes the open sentence true. If you are given a graph of a function, a solution is any point on that graph. The graph of any function, f, is the graph of the equation $y = f(x)$. If you have intersecting graphs, a solution is the ordered pair that satisfies both functions at the same time, or the **intersection point** of the graphs.

To determine how many minutes it takes for the plane to reach 7200 feet using your graph, you need to determine the intersection of the two graphs represented by the equation $7200 = 1800t$.

First, graph each side of the equation and then determine the intersection point of the two graphs.



$h(t) = 1800t$

$$7200 = 1800t$$

$\downarrow$ $\qquad$ $\downarrow$

$y = 7200$ $\qquad$ $y = 1800x$

Solution: (4, 7200)

After takeoff, it takes the plane 4 minutes to reach a height of 7200 feet.

3. What does $(t, h(t))$ represent?

4. Explain the connection between the form of the function $h(t) = 1800t$ and the equation $y = 1800x$ in terms of the independent and dependent quantities.

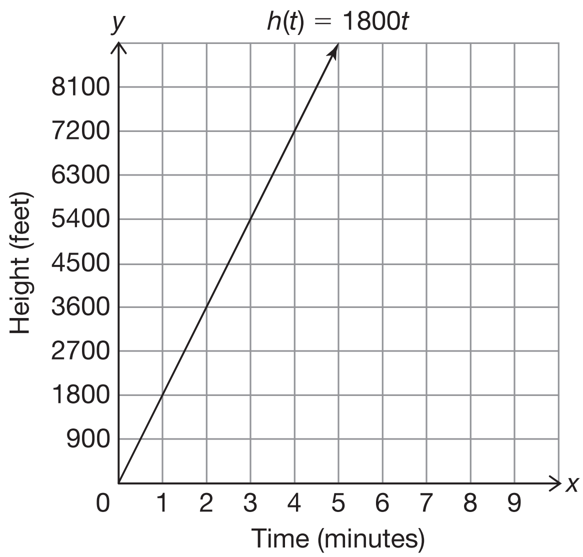
5. Use the graph to determine how many minutes it will take the plane to reach each height.

a. $h(t) = 5400$

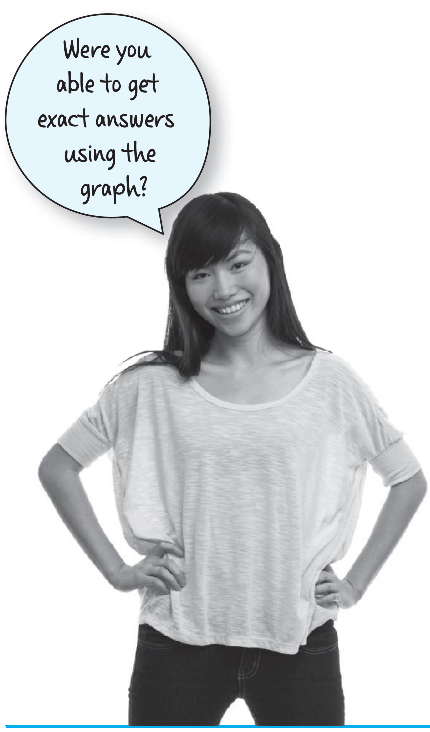
b. $h(t) = 9000$

c. $h(t) = 3150$

d. $h(t) = 4500$



6. Compare and contrast your solutions using the graphing method to the solutions in Question 2, parts (a) through (d) where you used an algebraic method. What do you notice?



Talk the Talk

You just worked with different representations of a linear function.

1. Describe how a linear function is represented:

   a. in a table.

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   b. in a graph.

   c. in an equation.

2. Name some advantages and disadvantages of the graphing method and the algebraic method when determining solutions for linear functions.

3. Do you think the graphing method for determining solutions will work for any function?

You solved several linear equations in this lesson. Remember, the Addition, Subtraction, Multiplication, and Division Properties of Equality allow you to balance and solve equations. The Distributive Property allows you to rewrite expressions to remove parentheses, and the Commutative and Associative Properties allow you to rearrange and regroup expressions.

4. Solve each equation and justify your reasoning.

a. $7x + 2 = -12$

b. $4(x + -7) + 12 = 20$

c. $14x - 13 = 9x + 1$

d. $\frac{x + 2}{6} = \frac{2}{5}$

Be prepared to share your solutions and methods.

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