2.2

What Goes Up Must Come Down

Analyzing Linear Functions

LEARNING GOALS

In this lesson, you will:

- Complete tables and graphs, and write equations to model linear situations.
- Analyze multiple representations of linear relationships.
- Identify units of measure associated with linear relationships.
- Determine solutions to linear functions using intersection points and properties of equality.
- Determine solutions using tables, graphs, and functions.
- Compare and contrast different problem-solving methods.
- Estimate solutions to linear functions.
- Use a graphing calculator to analyze functions and their graphs.

The dollar is just one example of currency used around the world. For example, Swedes use the krona, Cubans use the peso, and the Japanese use the yen. This means that if you travel to another country you will most likely need to exchange your U.S. dollars for a different currency. The exchange rate represents the value of one country's currency in terms of another—and it is changing all the time. In some countries, the U.S. dollar is worth more. In other countries, the dollar is not worth as much.

Why would knowing the currency of another country and the exchange rate be important when planning trips?

PROBLEM 1 As We Make Our Final Descent

At 36,000 feet, the crew aboard the 747 airplane begins making preparations to land. The plane descends at a rate of 1500 feet per minute until it lands.

1. Compare this problem situation to the problem situation in Lesson 2.1, *The Plane!* How are the situations the same? How are they different?

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2. Complete the table to represent this problem situation.

	Independent Quantity	Dependent Quantity
Quantity		
Units		
	0	
	2	
	4	
	6	
		18,000
		6000
Expression	t	

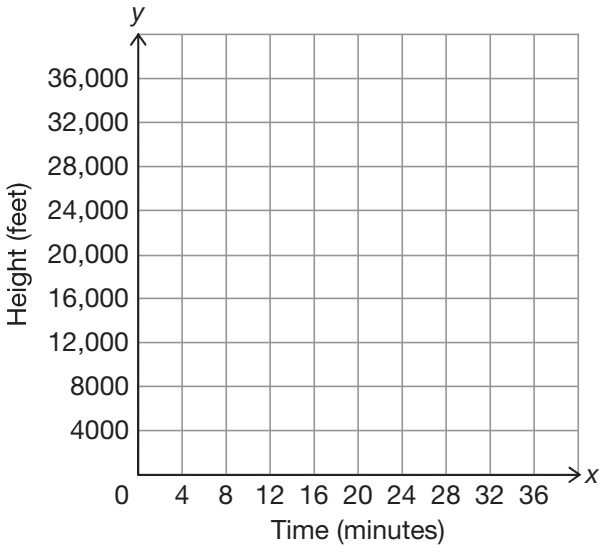
3. Write a function, $g(t)$, to represent this problem situation.

4. Complete the table shown. First, determine the unit of measure for each expression. Then, describe the contextual meaning of each part of the function. Finally, choose a term from the word box to describe the mathematical meaning of each part of the function.

input value	output value	rate of change
y-intercept	*x*-intercept	

		Description	
Expression	**Units**	**Contextual Meaning**	**Mathematical Meaning**
t			
-1500			
$-1500t$			
36,000			
$-1500t + 36{,}000$			

5. Graph $g(t)$ on the coordinate plane shown.



You have just represented the *As We Make Our Final Descent* scenario in different ways:

- numerically, by completing a table,
- algebraically, by writing a function, and
- graphically, by plotting points.

Let's consider how to use each of these representations to answer questions about the problem situation.

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6. Determine how long will it take the plane to descend to 14,000 feet.

a. Use the table to determine how long it will take the plane to descend to 14,000 feet.

b. Graph and label $y = 14{,}000$ on the coordinate plane. Then determine the intersection point. Explain what the intersection point means in terms of this problem situation.

c. Substitute 14,000 for $g(t)$ and solve the equation for t. Interpret your solution in terms of this problem situation.

d. Compare and contrast your solutions using the table, graph, and the function. What do you notice? Explain your reasoning.

7. Determine how long it will take the plane to descend to 24,000 feet.

   a. Use the table to determine how long it will take the plane to descend to 24,000 feet.

   b. Graph and label $y = 24{,}000$ on the coordinate plane. Then determine the intersection point. Explain what the intersection point means in terms of this situation.

   c. Substitute 24,000 for $g(t)$ and solve the equation for t. Interpret your solution in terms of this situation.

   d. Compare and contrast your solutions using the table, graph, and the function. What do you notice? Explain your reasoning.

8. For how many heights can you calculate the *exact* time using the:

a. table?

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b. graph?

c. function?

9. Use the word bank to complete each sentence.

always	sometimes	never

If I am given a dependent value and need to calculate an independent value of a linear function,

a. I can _____ use a table to determine an *approximate* value.

b. I can _____ use a table to calculate an *exact* value.

c. I can _____ use a graph to determine an *approximate* value.

d. I can _____ use a graph to calculate an *exact* value.

e. I can _____ use a function to determine an *approximate* value.

f. I can _____ use a function to calculate an *exact* value.

PROBLEM 2 Making the Exchange

The plane has landed in the United Kingdom and the Foreign Language Club is ready for their adventure. Each student on the trip boarded the plane with £300. They each brought additional U.S. dollars with them to exchange as needed. The exchange rate from U.S. dollars to British pounds is £0.622101 pound to every dollar.

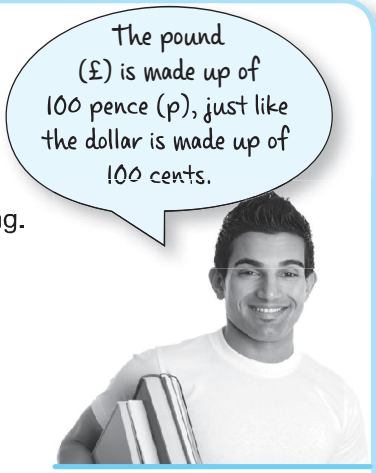
1. Write a function to represent the total amount of money in British pounds each student will have after exchanging additional U.S. currency. Define your variables.

2. Identify the slope and interpret its meaning in terms of this problem situation.

3. Identify the *y*-intercept and interpret its meaning in terms of this problem situation.

4. Dawson would like to exchange $70 more.

Jonathon thinks Dawson should have a total of £343.54707. Erin says he should have a total of £343.55, and Tre says he should have a total of £342. Who's correct? Who's reasoning is correct? Why are the other students not correct? Explain your reasoning.



Jonathon

$£(d) = 300 + 0.622101d$

$£(d) = 300 + 0.622101(70)$

$£(d) = 300 + 43.54707$

$£(d) = 343.54707$

Erin

$f(d) = 300 + 0.622101d$

$f(d) = 300 + 0.622101(70)$

$f(d) = 300 + 43.54707$

$f(d) = 343.54707$

$f(d) \approx 343.55$

Tre

$f(d) = 300 + 0.6d$

$f(d) = 300 + 0.6\,(70)$

$f(d) = 300 + 42$

$f(d) = 342$

5. How many total pounds will Dawson have if he only exchanges an additional $50? Show your work.

PROBLEM 3 Using Technology to Complete Tables

Throughout this lesson you used multiple representations and paper-and-pencil to answer questions. You can also use a graphing calculator to answer questions. Let's first explore how to use a graphing calculator to create a table of values for converting U.S. dollars to British pounds.

The exchange function is $f(d) = 300 + 0.622101d$.

You can use a graphing calculator to complete a table of values for a given function.

Step 1: Press **Y=**

Step 2: Enter the function. Press **ENTER**.

Step 3: Press **2ND TBLSET** (above **WINDOW**).

TblStart is the starting data value for your table. Enter this value.

ΔTbl (read "delta table") is the increment. This value tells the table what intervals to count by for the independent quantity. If **ΔTbl** = 1 then the values in your table would go up by 1s. If **ΔTbl** = -1, the values would go *down* by 1s. Enter the **ΔTbl**.

For this scenario, you will not exchange any currency less than \$100. Set the **TblStart** value to 100.

Step 4: Press **2ND TABLE** (above **GRAPH**). Use the up and down arrows to scroll through the data.

1. Use your graphing calculator and the **TABLE** feature to complete the table shown.

U.S. Currency	British Currency
$	£
100	
150	
175	
	455.53
	466.10

Analyze the given U.S. Currency dollar amounts and decide how to set the increments for ΔTbl.

2. Were you able to complete the table using the **TABLE** feature? Why or why not? What adjustments, if any, can you make to complete the table?

2

PROBLEM 4 Using Technology to Analyze Graphs

There are several graphing calculator strategies you can use to analyze graphs to answer questions. Let's first explore the **value** feature. This feature works well when you are given an independent value and want to determine the corresponding dependent value.

You can use the **value** feature on a graphing calculator to determine an exact data value on a graph.

Step 1: Press **Y=**. Enter your function.

Step 2: Press **WINDOW**. Set appropriate values for your function. Then press **GRAPH**.

Step 3: Press **2ND** and then **CALC**. Select **1:value**. Press **ENTER**. Then type the given independent value next to **X=** and press **ENTER**. The cursor moves to the given independent value and the corresponding dependent value is displayed at the bottom of the screen.

Be sure to double check that you typed in the correct function.

If you get an error message, go back and adjust your WINDOW.
ERR:INVALID
1: Quit
2: Goto

Use the **value** feature to answer each question.

1. How many total British pounds will Amy have if she exchanges an additional:
   a. $375?

   b. $650?

   c. $2000

2. How can you verify that each solution is correct?

3. What are the advantages and limitations of using the **value** feature?

Let's now explore the **intersect** feature of **CALC**. You can use this feature to determine an independent value when given a dependent value.

Suppose you know that Jorge has a total of £725.35. You can first write this as $f(d) = 300 + 0.622101d$ and $y = 725.35$. Then graph each equation, calculate the intersection point, and determine the additional amount of U.S. currency that Jorge exchanged.

You can use the **intersect** feature to determine an independent value when given a dependent value.

Step 1: Press **Y=**. Enter the two equations, one next to Y_1= and one next to Y_2=.

Step 2: Press **WINDOW**. Set appropriate bounds so you can see the intersection of the two equations. Then press **GRAPH**.

Step 3: Press **2ND CALC** and then select **5:intersect**. The cursor should appear somewhere on one of the graphs, and at the bottom of the screen you will see **First curve?** Press **ENTER**.
The cursor should then move to somewhere on the other graph, and you will see **Second curve?** Press **ENTER**.
You will see **Guess?** at the bottom of the screen. Move the cursor to where you think the intersection point is and Press **ENTER**.
The intersection point will appear.

You can use your arrow keys to scroll to different features.

Use the **intersect** feature to answer each question.

4. How many additional U.S. dollars did Jorge exchange if he has a total of:

a. £725.35?

b. £1699.73?

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5. How can you verify that each solution is correct?

6. What are the advantages and limitations of the intersect feature?

7. Do you think you could use each of the graphing calculator strategies discussed in this lesson with any function, not just linear functions?

PROBLEM 5 Graphing Calculator Practice

Use a graphing calculator to evaluate each function. Explain the strategy you used.

1. $f(x) = 14.95x + 31.6$

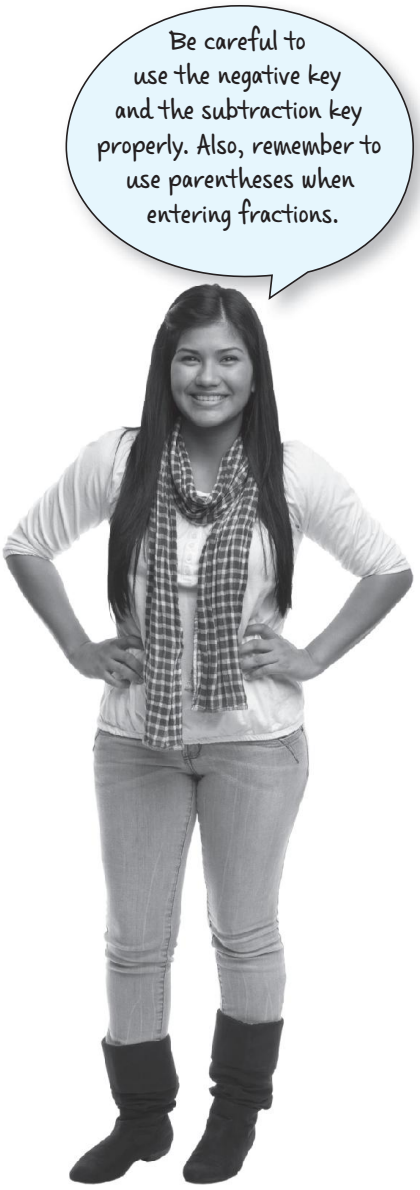
a. $f(3.5)$

b. $f(16.37)$

c. $f(50.1)$

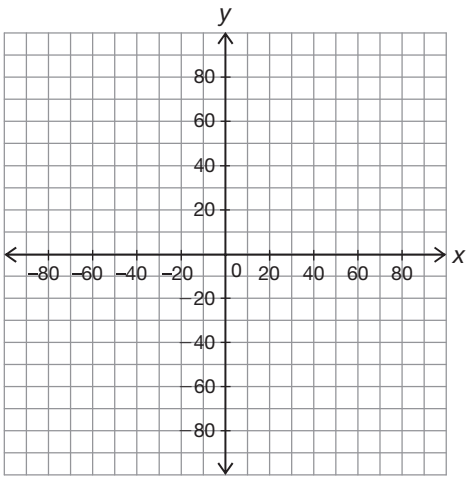
2.

x	$-\frac{7}{9}x - 18$
$-\frac{1}{2}$	
0	
$-4\frac{1}{2}$	

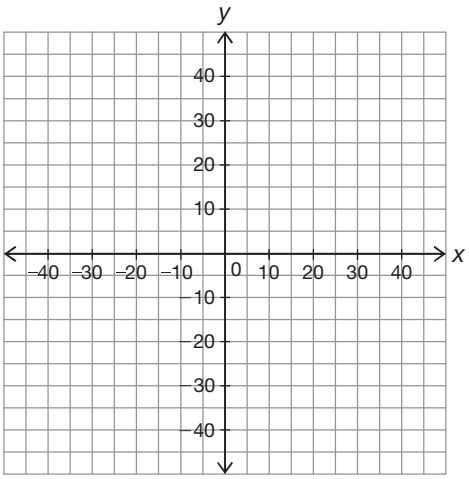


3. Use a graphing calculator and the **intersect** feature to determine each independent value. Then sketch the graphs on the coordinate plane provided.

a. $f(x) = -3.315x - 20$ when $f(x) = 23.38$



b. $\frac{1}{2}x + 5 = 16\frac{4}{5}$



4. Use the word box to complete each sentence, and then explain your reasoning.

always	sometimes	never

If I am using a graphing calculator and I am given a dependent value and need to calculate an independent value,

a. I can _____ use a table to determine an *approximate* value.

b. I can _____ use a table to calculate an *exact* value.

c. I can _____ use a graph to determine an *approximate* value.

d. I can _____ use a graph to calculate an exact value.

e. I can _____ use a function to determine an approximate value.

f. I can _____ use a function to calculate an exact value.

Be prepared to share your solutions and methods.