2.5

Play Ball!

Absolute Value Equations and Inequalities

LEARNING GOALS

In this lesson, you will:

- Understand and solve absolute values.
- Solve linear absolute value equations.
- Solve and graph linear absolute value inequalities on number lines.
- Graph linear absolute values and use the graph to determine solutions.

KEY TERMS

- opposites
- absolute value
- linear absolute value equation
- linear absolute value inequality
- equivalent compound inequalities

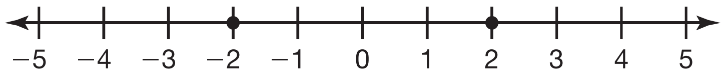
All games and sports have specific rules and regulations. There are rules about how many points each score is worth, what is in-bounds and what is out-of-bounds, and what is considered a penalty. These rules are usually obvious to anyone who watches a game. However, some of the regulations are not so obvious. For example, the National Hockey League created a rule that states that a blade of a hockey stick cannot be more than three inches or less than two inches in width at any point. In the National Football League, teams that wear black shoes must wear black shoelaces and teams that wear white shoes must wear white laces. In the National Basketball Association, the rim of the basket must be a circle exactly 18 inches in diameter. Most sports even have rules about how large the numbers on a player's jersey can be!

Do you think all these rules and regulations are important? Does it really matter what color a player's shoelaces are? Why do you think professional sports have these rules, and how might the sport be different if these rules did not exist?

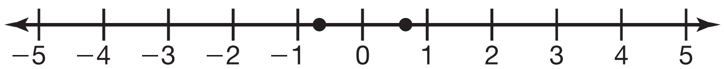
PROBLEM 1 Opposites Attract? Absolutely!

1. Analyze each pair of numbers and the corresponding graph.

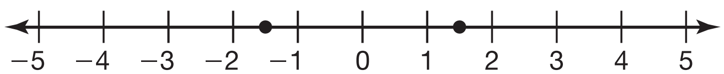
   a. -2 and 2



   b. $-\frac{2}{3}$ and $\frac{2}{3}$



   c. -1.5 and 1.5



2. Describe the relationship between the two numbers.

3. What do you notice about the distance each point lies away from zero on each number line?

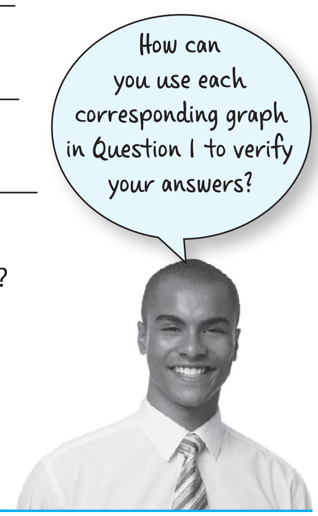
Two numbers that are an equal distance, but are in different directions, from zero on the number line are called **opposites**. The **absolute value** of a number is its distance from zero on the number line.

4. Write each absolute value.

   a. $|-2| =$ ________ $\qquad |2| =$ ________

   b. $\left|-\frac{2}{3}\right| =$ ________ $\qquad \left|\frac{2}{3}\right| =$ ________

   c. $|-1.5| =$ ________ $\qquad |1.5| =$ ________



5. What do you notice about each set of answers for Question 4?

6. Determine the value of each. Show your work.

a. $|3 - 8|$

b. $|3| - |8|$

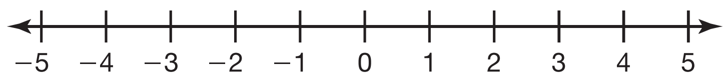
c. $|4(5)|$

d. $|-4| \cdot |5|$

e. $\left|\frac{12}{-3}\right|$

f. $\frac{|12|}{|-3|}$

7. Determine the solution(s) to each equation.



a. $x = 5$

b. $|x| = 5$

c. $|x| = -5$

d. $|x| = 0$

8. Analyze each equation containing an absolute value symbol in Question 7. What does the form of the equation tell you about the possible number of solutions?

PROBLEM 2 Too Heavy? Too Light? You're Out!

The official rules of baseball state that all baseballs used during professional games must be within a specified range of weights. The baseball manufacturer sets the target weight of the balls at 145.045 grams on its machines. The specified weight allows for a difference of 3.295 grams. This means the weight can be 3.295 grams greater than or less than the target weight.

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1. Write an expression to represent the difference between a manufactured baseball's weight and the target weight. Use w to represent a manufactured baseball's weight.

2. Suppose the manufactured baseball has a weight that is greater than the target weight.
   a. Write an equation to represent the greatest acceptable difference in the weight of a baseball.

   b. Solve your equation to determine the greatest acceptable weight of a baseball.

3. Suppose the manufactured baseball has a weight that is less than the target weight.
   a. Write an equation to represent the least acceptable difference in weight.

   b. Solve your equation to determine the least acceptable weight of a baseball.

The two equations you wrote can be represented by the **linear absolute value equation** $|w - 145.045| = 3.295$. In order to solve any absolute value equation, recall the definition of absolute value.

Consider this linear absolute value equation.

$$|a| = 6$$

There are two points that are 6 units away from zero on the number line: one to the right of zero, and one to the left of zero.

$$+(a) = 6 \quad \text{or} \quad -(a) = 6$$

$$a = 6 \quad \text{or} \quad a = -6$$

Now consider the case where $a = x - 1$.

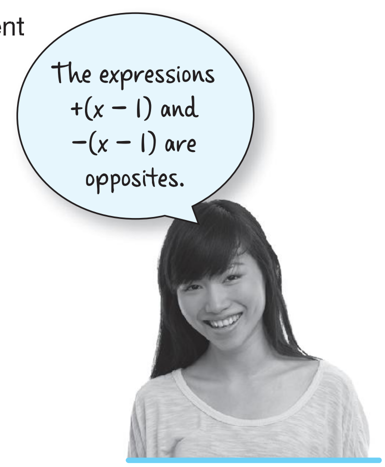
$$|x - 1| = 6$$

If you know that $|a| = 6$ can be written as two separate equations, you can rewrite any absolute value equation.

$$+(a) = 6 \quad \text{or} \quad -(a) = 6$$

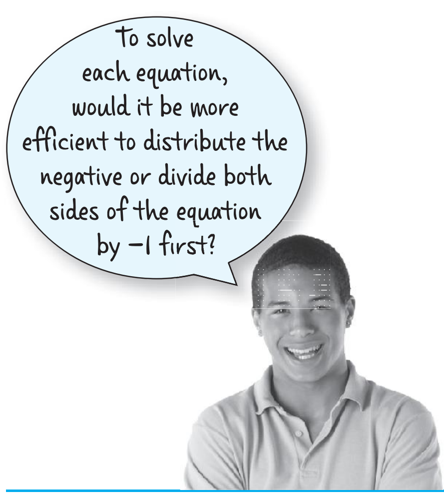
$$+(x - 1) = 6 \quad \text{or} \quad -(x - 1) = 6$$

4. How do you know the expressions $+(a)$ and $-(a)$ represent opposite distances?



5. Determine the solution(s) to the linear absolute value equation $|x - 1| = 6$. Then check your answer.

$+(x - 1) = 6$ $\qquad\qquad$ $-(x - 1) = 6$



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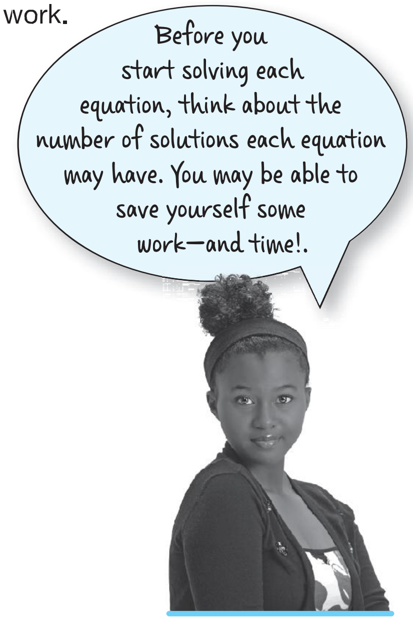
6. Solve each linear absolute value equation. Show your work.



a. $|x + 7| = 3$

b. $|x - 9| = 12$

c. $|3x + 7| = -8$

d. $|2x + 3| = 0$

7. Cho, Steve, Artie, and Donald each solved the equation $|x| - 4 = 5$.

Artie

$|x| - 4 = 5$

$(x) - 4 = 5$ $\quad$ $-(x) - 4 = 5$

$(x) = 9$ $\quad$ $-x = 9$

$x = -9$

Donald

$|x| - 4 = 5$

$|x| = 9$

$(x) = 9$ $\quad$ $-(x) = 9$

$x = -9$

Cho

$|x| - 4 = 5$

$(x) - 4 = 5$ $\quad$ $-[(x) - 4] = 5$

$x - 4 = 5$ $\quad$ $-x + 4 = 5$

$x = 9$ $\quad$ $-x = 1$

$x = -1$

Steve

$|x| - 4 = 5$

$(x) - 4 = +5$ $\quad$ $-(x) - 4 = -5$

$x = 9$ $\quad$ $-x - 4 = -5$

$-x = -1$

$x = 1$

a. Explain how Cho and Steve incorrectly rewrote the absolute value equation as two separate equations.

b. Explain the difference in the strategies that Artie and Donald used. Which strategy do you prefer? Why?

8. Solve each linear absolute value equation.

a. $|x| + 16 = 32$

b. $23 = |x - 8| + 6$

c. $3|x - 2| = 12$

d. $35 = 5|x + 6| - 10$