Prepping for the Robot Challenge

6.1

Key

Solving Linear Systems Graphically and Algebraically

LEARNING GOALS

In this lesson, you will:

- Write systems of linear equations.
- Graph systems of linear equations.
- Determine the intersection point, or break-even point, from a graph.
- Use the substitution method to determine the intersection point.
- Understand that systems of equations can have one, zero, or infinite solutions.

KEY TERMS

- break-even point
- system of linear equations
- substitution method
- consistent systems
- inconsistent systems

In today's world, the field of robotics is growing rapidly; however the interest and fascination with robots has been occurring for centuries. The word *robot* was first used in a 1920 play describing a factory that creates artificial people which could be mistaken for humans. However, the idea of *robots* did not begin there.

Descriptions of some of the first automatons, or self-operating machines, were recorded as early as the 3rd century BC! An ancient Chinese text describes a mechanical engineer presenting King Mu of Zhou with a life-size, human-shaped figure that could walk, move its head, and sing. By the 1200s, a Muslim engineer named al-Jazari created some of the first human-like machines that could be used for practical purposes. He created a drink-serving waitress and a hand-washing automaton that were both functional using hydropower. It is amazing to think of these people inventing such incredible robots without the use of today's technology.

Are you surprised to learn that people created robots so long ago? What do we use robots for today? What do you think robots will be able to do in the future?

PROBLEM 1 Gearing For Success

Gwen has a part-time job working at Reliable Robots (RR) which sells electronics and hardware parts for robot creators. One of her tasks is to analyze RR's finances in terms of cost and income. Her boss, Mr. Robo, asks her to determine the *break-even point* for the cost and the income. The **break-even point** is the point when the cost and the income are equal. Gwen begins with the income and costs for gearboxes.

1. Let the function $I(g)$ represent the income (I) from selling gearboxes (g) and the function $C(g)$ represent the cost (C) of purchasing gearboxes (g).

   a. Describe the relationship between the income function and the cost function that will show the break-even point. Explain your reasoning.

   The break-even point will occur when the cost function, C(g), is equal to the income function, (g). when C(g) = I(g), money is not lost, nor is money gained, the company breaks-even.

   b. Describe the relationship between the income function and the cost function that will show a profit from selling gearboxes. Explain your reasoning.

   A profit will be made when the income earned is greater than the costs incurred. This happens when I(g) > C(g). When there is more income than costs, the company will make a profit.

2. RR purchases gearboxes from The Metalists for \$5.77 per gearbox plus a one-time credit check fee of \$45.00. RR sells each gearbox for \$8.50.

   a. Write the function for the income generated from selling gearboxes.

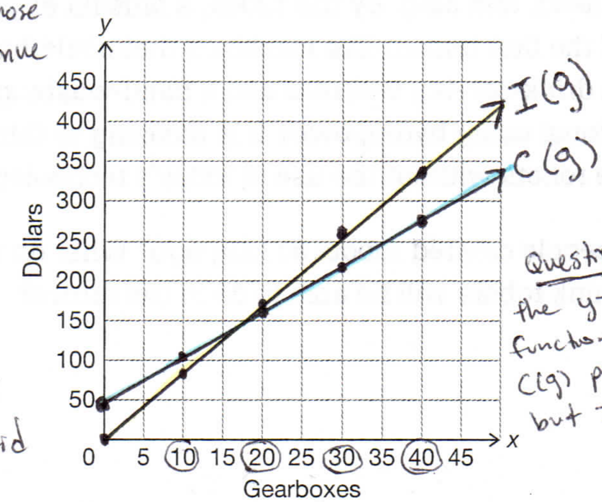
   $\boxed{I(g) = 8.50g}$ "The income from selling gearboxes is \$8.50 per gear box"

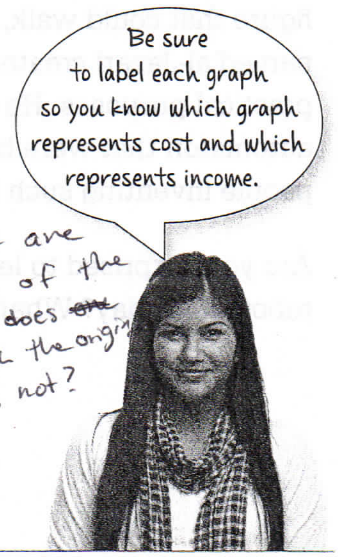
   b. Write the function for the cost of purchasing gearboxes from The Metalists.

   $\boxed{C(g) = 45 + 5.77g}$ "The cost of selling gearboxes is the \$45 credit check, plus \$5.77 per gearbox"

3. Sketch a graph of each function on the coordinate plane to predict the break-even point of the income from RR selling the gearboxes and the cost of purchasing the gearboxes.

Note: Mr Marcus chose to find the revenue and costs of making/selling 10, 20, 30, 40 gearboxes istead of 5, 15, 25, 35, 45. Why? Why did he not plot all of the points? What did you do? Why?





Question: What are the y-intercepts of the functions? Why does C(g) pass through the origin but I(g) does not?

a. How is the break-even point for $I(g)$ and $C(g)$ represented on the graph you sketched? Estimate the break-even point.

The break-even point is represented on the graph where the two lines intersect. Where the functions intersect is where the functions are equal, and when Income = Costs is where the break-even point can be found, which is approximately 17 gear boxes

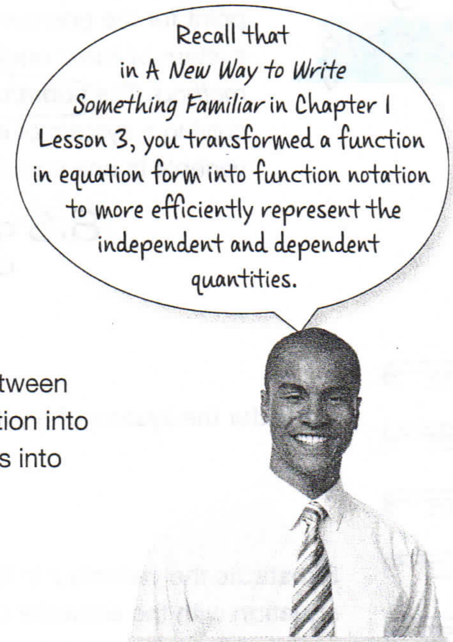
b. Could you determine the exact break-even point from the graph? Why or why not.

No, we could not determine the exact break-even point from the graph because the lines do not cross at a perfect intersection point on the graph. We can tell they intersect between 15 and 20 gear boxes.

As you learned previously, the coordinates of an intersection point of two graphs can be exact or approximate depending on whether the intersection point is located on the intersection of two grid lines. You also learned that you had to use algebra to prove an exact intersection point.

Notice the units of measure for the independent and dependent quantities in $I(g)$ and $C(g)$ are the same.

In both functions $I(g)$ and $C(g)$, the g represents the independent quantity, gearboxes, and the dependent quantity is dollars. However, the dollars in each function represent different "types" of dollars. You know that dollars are represented differently because you determined two different functions—one function for cost (in dollars) and one function for income (in dollars).



Notes

When determining the break-even point algebraically between two functions, it is more efficient to transform each function into equation form. In this case, by transforming the functions into equation form, you establish one unit of measure for the dependent quantity: dollars.

Two ways to write the same equations

Analyze the functions representing cost and income from gearboxes for Reliable Robots.

function notation $\qquad I(g) = 8.5g \qquad C(g) = 5.77g + 45$

Since g is the independent variable, you can represent g as x in equation form.

$$I(x) = 8.5x \qquad C(x) = 5.77x + 45$$

Since both $I(g)$ and $C(g)$ represent the dependent quantity in dollars, you can represent each using y as the variable.

Equation form $\qquad y = 8.5x \qquad y = 5.77x + 45$

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4. Do you think it is possible to use other variables instead of x and y when transforming a function written in function notation to equation form?

Yes, you can use other variables, but x and y are more common.

When two or more equations define a relationship between quantities, they form a **system of linear equations.**

5. What is the relationship between the two equations in this problem situation?

We are looking at when are the two equations the same, so the two equations are equal

$I(g) = C(g)$

$I(g) = 8.5g$

$C(g) = 45 + 5.77g$

Now that you have successfully created a system of linear equations, you can determine the break-even point for the gearboxes at RR. One way to solve a system of linear equations is called the *substitution method*. The **substitution method** is a process of solving a system of equations by substituting a variable in one equation with an equivalent expression.

$8.5g = 45 + 5.77g$ or $8.5x = 45 + 5.77x$

Consider the system of equations from the previous worked example.

$$\begin{cases} y = 5.77x + 45 \\ y = 8.5x \end{cases}$$

Substitute the variable y in the first equation with the equivalent expression in the second equation. $8.5x = 5.77x + 45$

Isolate the variable to solve. $2.73x = 45$

$x \approx 16.48$

Question: where did this come from? What does $x \approx 16.48$ mean in terms of our gearbox problem? What else can you now tell me?

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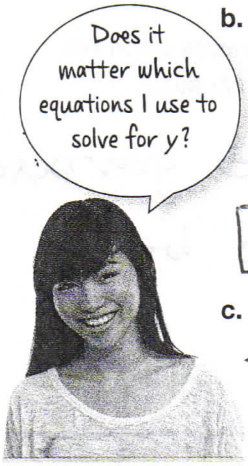
6. Analyze the solution $x \approx 16.48$.

a. What does this point represent in terms of the problem situation? Why is this solution an approximation?

This point represents the number of gearboxes that must be made, and sold, in order to break-even. The solution is approximate because it was rounded to the nearest hundredths place.

b. Solve for y. Describe the solution in terms of this problem situation.

Note: (2 options b/c there are 2 equations to use)



$y = 8.5x$ ← Income equation looks "easier" to work with

$y = 8.5(16.48)$ substitute 16.48 in for x to find out how much income is made from selling 16.48 gearboxes (this is the same as the cost of making 16.48 gear boxes)

$\boxed{y = 140.08}$ $140.08 is the cost and income from 16.48 gearboxes

c. What is the profit from gearboxes at the break-even point?

The profit from selling 16.48 gearboxes is $0 because the income is exactly the same as the costs (Income − Costs = Profit)

d. Does this break-even point make sense in terms of the problem situation? Why or why not.

The break-even point, (16.48, 140.08), does not make sense because you cannot sell .48 of a gearbox, you can only sell whole gearboxes, such as 16 or 17.

7. Analyze your graph of the cost and the income for the different number of gearboxes.

a. Draw a box around the portion of the graph that represents when RR is losing money. Then write an inequality to represent this portion of the graph and describe what it means in terms of the problem situation.

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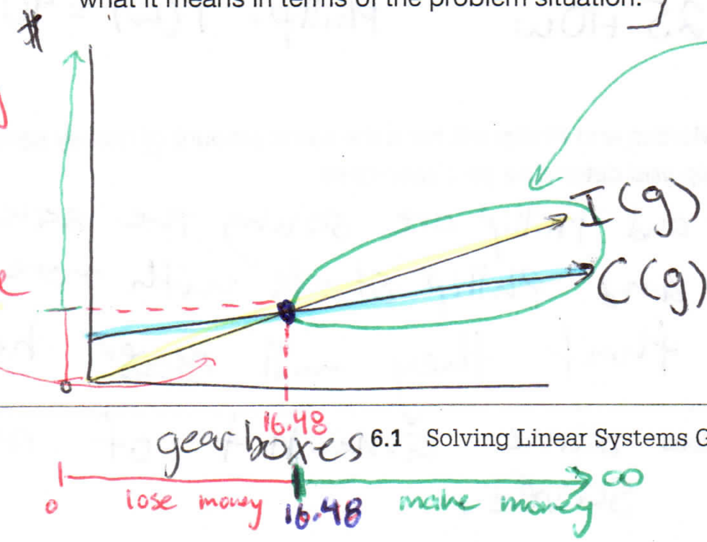
sketch

losing money when costs are higher than income

making money when income, I(g), is greater than costs



$\boxed{g < 17}$

see graph on prior page

b. Draw an oval around the portion of the graph that represents when RR is earning money. Then write an inequality to represent this portion of the graph and describe what it means in terms of the problem situation.

RR is making a profit when they sell more than 16.48 gearboxes (17 gearboxes more).

$\boxed{g \geq 17}$

c. Write an equation to represent the portion of the graph that represents when RR breaks even and describe what it means in terms of the problem situation.

RR breaks-even when they sell/make 16.48 gearboxes, which is impossible, but the equation would be

$\boxed{g = 16.48}$

PROBLEM 2 Saving Up

Marcus and Phillip are in the Robotics Club. They are both saving money to buy materials to build a new robot. They plan to save the same amount of money each week.

1. Write a function to represent the time it takes Marcus and Phillip to save money. Define your variables and explain why you chose those variables.

$S(w)$ = amount of saving after, w, weeks

w = # of weeks

a = \$ per week that is saved

$\boxed{S(w) = a \cdot w}$

Marcus decides to open a new bank account. He deposits \$25 that he won in a robotics competition. He also plans on depositing \$10 a week that he earns from tutoring. Phillip decides he wants to keep his money in a sock drawer. He already has \$40 saved from mowing lawns over the summer. He plans to also save \$10 a week from his allowance.

2. Write a function to represent the information regarding Marcus and Phillip saving money for new robotics materials.

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Marcus: $M(w) = 25 + 10w$ Phillip: $P(w) = 40 + 10w$

3. Predict when Marcus and Phillip will have the same amount of money saved. Use your functions to help you determine your prediction.

Since Marcus and Phillip are saving the same amount of money each week, and Phillip starts with more money initially, that means that they will never have the same amount of money saved.

when does $M(w) = P(w)$?

You can prove your prediction by solving and graphing a system of linear equations.

4. Rewrite each function as an equation. Use *x* and *y* for the variables of each function in equation form and define the variables. Then, write a system of linear equations.

$M(w) = P(w)$ set equations equal

$25 + 10w = 40 + 10w$ substitute

$-10w \quad -10w$

$\boxed{25 \neq 40}$ simplify

since $25 \neq 40$, that means that there is <u>no</u> w that will make the two equations equal

5. Analyze each equation.

   a. Describe what the slope of each line represents in this problem situation.

The slope is the amount of money they save <u>each</u> week.

   b. How do the slopes compare? Describe what this means in terms of this problem situation.

Marcus: \$10 per week → $\frac{10}{1}$

Phillip: \$10 per week → $\frac{10}{1}$

The slopes are the same, meaning they save the same amount each week.

   c. Describe what the *y*-intercept of each line represents in this problem situation.

The y-intercept of each line represents how much money each of them started with saving initially.

   d. How do the *y*-intercepts compare? Describe what this means in terms of this problem situation.

Marcus: 25

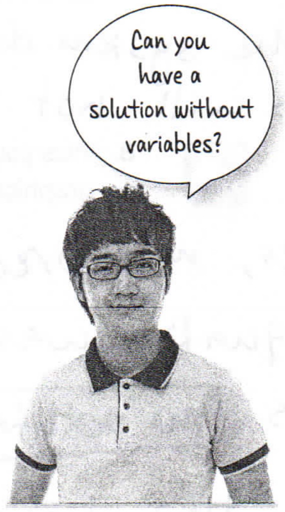
Phillip: 40

Phillip began with more money in his savings.

6. Determine the solution of the system of linear equations algebraically and graphically.

   a. Use the substitution method to determine the intersection point. see above in question 4

   b. Does your solution make sense? Describe what this means in terms of the problem situation.

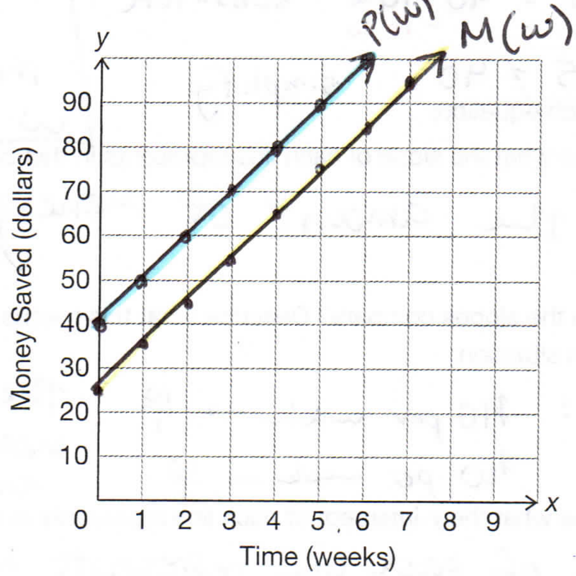


Phillip and Marcus will not have the same amount of money in their savings accounts.

c. Predict what the graph of this system will look like. Explain your reasoning.

The graphs will be parallel lines with a slope of 10.

d. Graph both equations on the coordinate plane provided.



7. Analyze the graph you created.

a. Describe the relationship between the graphs.

The graphs are parallel.

b. Does this linear system have a solution? Explain your reasoning.

No, the system does not have a solution because the two graphs will not intersect.

8. Was your prediction in Question 3 correct? Explain how you algebraically and graphically proved your prediction.

yes, my prediction was correct. Algebraically, the equation was false, $25 \neq 40$, which means there is $\boxed{\text{no solution}}$. Graphically, I could see that the two lines would never intersect, which means that Phillip's and Marcus' savings would never be equal.

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9. Tonya is also in the Robotics Club and has heard about Marcus's and Phillip's savings plans. She wants to be able to buy her new materials before Phillip, so she opens her own bank account. She is able to deposit $40 in her account that she has saved from her job as a waitress. Each week she also deposits $4 from her tips.

   a. Write a function that represents the information about Tonya saving money every week. Do not forget to define your variables.

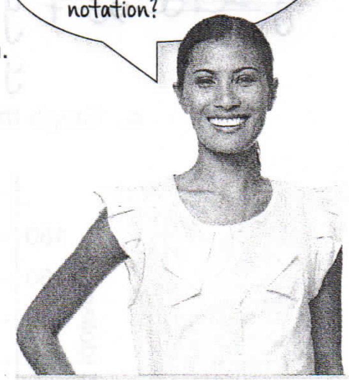
$T(w) = 40 + 4w$

   b. Write a linear system to represent the total amount of money Tonya and Phillip have after a certain amount of time.

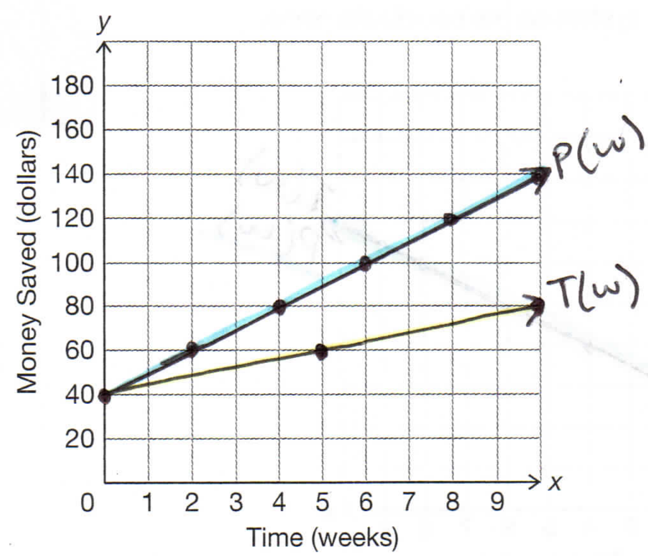
$T(w) = 40 + 4w$ $\quad y = 40 + 4x$

$P(w) = 40 + 10w$ $\quad y = 40 + 10x$

What is the most efficient way to write a system of linear equations if you have a function in function notation?



   c. Graph the linear system on the coordinate plane shown.



10. Do the graphs intersect? If so, describe the meaning in terms of this problem situation.

Yes, the graphs intersect. This means that Tonya and Phillip have the same amount of money saved, $40, initially.

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11. Phillip and Tonya went on a shopping spree this weekend and spent all their savings except for $40 each. Phillip is still saving $10 a week from his allowance. Tonya now deposits her tips twice a week. On Tuesdays she deposits $4 and on Saturdays she deposits $6. Phillip claims he is still saving more each week than Tonya.

   a. Do you think Phillip's claim is true? Explain your reasoning.

No, I do not think that Phillip's claim is true because they are both saving a total of $10 per week.

b. How can you prove your prediction?

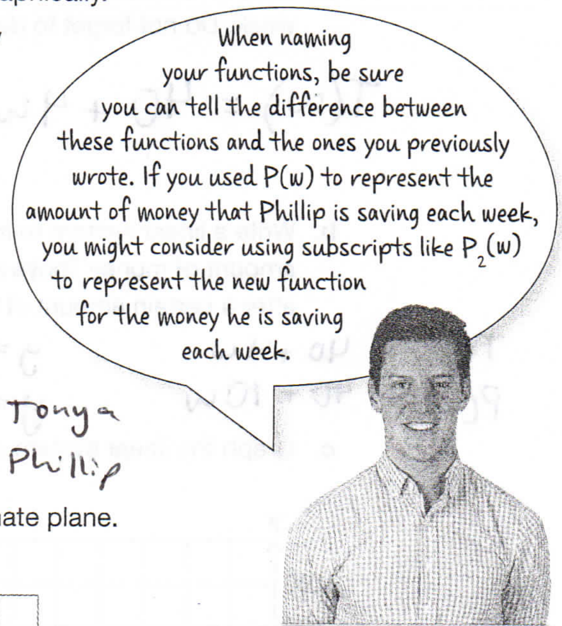
I could re-write the equations and simplify Tonya's to show they are the same.

12. Prove your prediction algebraically and graphically.

a. Write functions that represent any new information about the way Tonya and Phillip are now saving money.

Tonya : $T(w) = (4+6) \cdot w + 40$

Phillip : $P(w) = 10 \cdot w + 40$

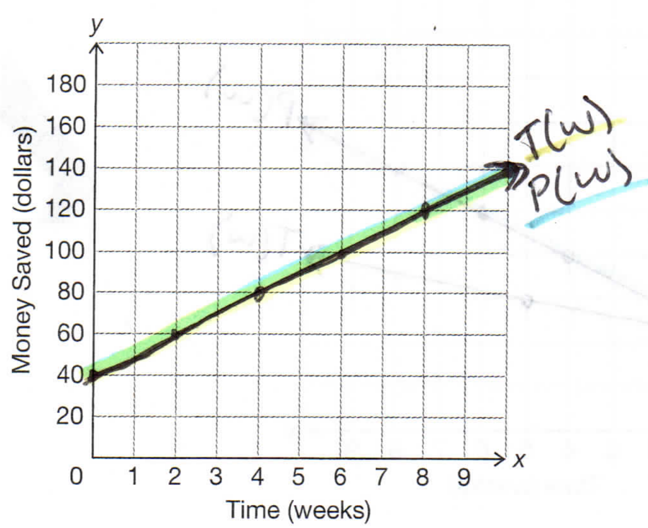


b. Write a new linear system to represent the total amount of money each friend has after a certain amount of time.

$y = (4+6) \cdot x + 40$ Tonya

$y = 10 \cdot x + 40$ Phillip

c. Graph the linear system on the coordinate plane.



13. Analyze the graph.

a. Describe the relationship between the graphs. What does this mean in terms of this problem situation?

The two graphs are identical, which means that Tonya and Phillip will always have the same amount of mony saved up.

b. Algebraically prove the relationship you stated in part (a).

$T(w) = P(w)$ set-up

$10w + 40 = 10w + 40$ substitute

$-10w$ $-10w$

$40 = 40$, so since that 40 always equals 40, that means that the two equations will always be equal.

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c. Does this solution prove the relationship? Explain your reasoning.

Yes, the solution does prove this relationship because 40 always equals 40, no matter what w equals.

14. Was Phillip's claim that he is still saving more than Tonya a true statement? Explain why or why not.

No, Phillip's claim was not correct because Tonya was also saving $10 per month, which was the same amount that Phillip was saving.

PROBLEM 3 Transforming Equations: More Than Meets the Eye

Not all systems will be written in slope-intercept form or function notation. Systems can also be written in standard form. Let's explore a system in standard form.

$$\begin{cases} 2x + 8y = 10 \\ 4x = y - 2 \end{cases}$$

Do you think there is more than one way to transform one of the equations in the system to create a new equation with only one unknown?

1. Analyze each student's work.

Dontrell

$$\begin{cases} 2x + 8y = 10 \\ 4x = y - 2 \end{cases}$$

$$2x + 8y = 10$$

$$4x + 2 = y$$

$$2x + 8(4x + 2) = 10$$

Janelle

$$\begin{cases} 2x + 8y = 10 \\ 4x = y - 2 \end{cases}$$

$$2x = 10 - 8y$$

$$x = 5 - 4y$$

$$4(5 - 4y) = y - 2$$

Maria

$$\begin{cases} 2x + 8y = 10 \\ 4x = y - 2 \end{cases}$$

$$8y = -2x + 10 \qquad 4x = y - 2$$

$$y = -\frac{2}{8}x + \frac{10}{8} \qquad 4x + 2 = y$$

$$-\frac{2}{8}x + \frac{10}{8} = 4x + 2$$

a. Describe the method Dontrell used to solve this system of equations and explain why he is correct.

Dontrell found out what y equaled, then substituted the value of y into the other equation for y.

b. Describe the method Janelle used to solve this system of equations and explain why her reasoning is correct.

Janelle solved the second equation for x, then substituted the value of x into the other equation for x.

c. Describe the method Maria used to solve this system of equations and explain why her reasoning is correct.

Maria solved both equations for y, then set each equation equal to eachother because y = y.

2. Which method do you prefer for solving this system of equations?

I prefer Dontrell's method because it appears to be a little easier. However, Maria's method makes the most sense to me.

3. Use one of the methods shown or use your own method to determine the solution to this system of equations.

Maria's Method:

① $8\cdot\left(-\frac{2}{8}x+\frac{10}{8}\right)=(4x+2)\cdot 8$ multiply both sides by 8

② $-2x+10=32x+16$ subtract 32x from both sides

$\quad -32x \qquad\quad -32x$

③ $-34x+10=16$ subtract 10 from each side

$\quad -10 \qquad -10$

6 ④ $\frac{-34x}{-34}=\frac{6}{-34}$ divide by −34

⑤ $x=\frac{-6}{34}$ simplify

$x=\frac{-3}{17}$

use either equation to find y

⑥ $y=4x+2$ substitute to find y

(continued on next page)

$x = \frac{-3}{17}$

(continued from prev. page)

$y = 4x + 2$ substitute

$y = 4\left(\frac{-3}{17}\right) + 2$ multiply $4\left(\frac{-3}{17}\right) \rightarrow \frac{-12}{17}$

$y = \frac{-12}{17} + \frac{34}{17}$ make common denominators

$2 \rightarrow \frac{2}{1} \rightarrow \frac{2 \cdot 17}{17} \rightarrow \frac{34}{17}$

$y = \frac{22}{17}$ add fractions

$\frac{-12}{17} + \frac{34}{17} = \frac{-12+34}{17} = \frac{22}{17}$

The solution to the system is

$\left(-\frac{3}{17}, \frac{22}{17}\right)$ or $(-0.18, 1.29)$

* The two graph intersect at the point $\left(\frac{-3}{17}, \frac{22}{17}\right)$.
If you plugged in $\frac{-3}{17}$ for x in both equations, you would get $\frac{22}{17}$ out for y in both equations

$y = -\frac{2}{8}x + \frac{10}{8}$

$y = -\frac{2}{8}\left(\frac{-3}{17}\right) + \frac{10}{8}$

$y = \frac{6}{136} + \frac{10}{8}$

$y = \frac{6}{136} + \frac{170}{136}$

$y = \frac{176}{136} = \frac{88}{68} = \frac{44}{34} = \frac{22}{17}$

$y = 4x + 2$

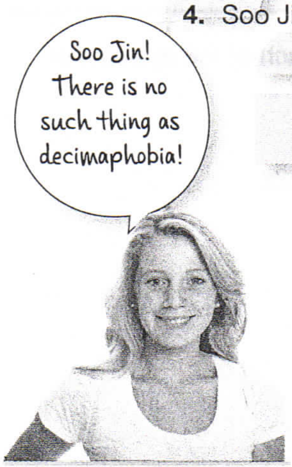
$y = 4\left(\frac{-3}{17}\right) + 2$

$y = \frac{-12}{17} + 2$

$y = \frac{-12}{17} + \frac{34}{17}$

$y = \frac{22}{17}$

when $x = \frac{-3}{17}$, y is $\frac{22}{17}$ for both equations



4. Soo Jin encountered this system of linear equations.

$$\begin{cases} 3.5x + 1.2y = 8 \\ 4.7x + 0.3y = 10.3 \end{cases}$$

However, Soo Jin has decimaphobia—a fear of decimals! Sammy tells her she has nothing to fear. He says, "All you need to do is multiply each equation by 10 to transform the system into whole numbers."

a. Is Sammy correct? Explain why or why not.

Yes, Sammy is correct because as long as you multiply both sides by 10, the equations will remain ballanced and equivalent. Both sides will be 10 times bigger.

b. Soo Jin attempts Sammy's method. Her work is shown.

Soo Jin

$$\begin{cases} 35x + 12y = 8 \\ 47x + 3y = 103 \end{cases}$$

forgot to multiply the right side by 10 as well.

Explain the mistake(s) Soo Jin made and then determine the correct way to rewrite this system.

$$\begin{cases} 35x + 12y = 80 \\ 47x + 3y = 103 \end{cases}$$

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Talk the Talk

1. Use any method of substitution to determine the solutions for each of the systems of linear equations.

a. $\begin{cases} 8x - 2y = 7 \\ 2x + y = 4 \end{cases}$ $\boxed{\left(\frac{5}{4}, \frac{3}{2}\right)}$

b. $\begin{cases} 0.4x + 0.3y = 1 \\ 0.1y = 0.2x \end{cases}$

$8x - 2y = 7$

$-2y = -8x + 7$

$y = 4x - \frac{7}{2}$

$2x + y = 4$

$y = -2x + 4$

$4x - \frac{7}{2} = -2x + 4$

$6x = \frac{7}{2} + \frac{8}{2}$

$6x = \frac{15}{2}$

$x = \frac{15}{2} \cdot \frac{1}{6}$

$x = \frac{15}{12} = \frac{5}{4}$

$y = -2x + 4$

$y = -2\left(\frac{5}{4}\right) + 4$

$y = \frac{-10}{4} + 4$

$y = \frac{-10}{4} + \frac{16}{4}$

$y = \frac{6}{4} = y = \frac{3}{2}$

c. $\begin{cases} \frac{1}{2}x + \frac{1}{4}y = 6 \\ y = 4 \end{cases}$

d. $\begin{cases} 6x + 3y = 5 \\ y = -2x + 1 \end{cases}$

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