There's Another Way?

Using Linear Combinations to Solve a Linear System

6.2

LEARNING GOALS

In this lesson, you will:

- Write a system of equations to represent a problem context.
- Solve a system of equations algebraically using linear combinations (elimination).

KEY TERM

- linear combinations method

Key

Morse code is a communication system which allows people to "speak" with sound. Words are transmitted using short sounds called "dits," which are represented in writing as dots; while long sounds, called "dahs," are represented in writing as dashes. The letters of the alphabet and digits each have their own unique collection of dits and dahs:

A	• —	U	• • —
B	— • • •	V	• • • —
C	— • — •	W	• — —
D	— • •	X	— • • —
E	•	Y	— • — —
F	• • — •	Z	— — • •
G	— — •		
H	• • • •		
I	• •		
J	• — — —		
K	— • —	1	• — — — —
L	• — • •	2	• • — — —
M	— —	3	• • • — —
N	— •	4	• • • • —
O	— — —	5	• • • • •
P	• — — •	6	— • • • •
Q	— — • —	7	— — • • •
R	• — •	8	— — — • •
S	• • •	9	— — — — •
T	—	0	— — — — —

When you combine these codes, you can produce sentences in Morse code.

Try it out! Communicate with your friends using Morse code.

PROBLEM 1 People Love Their Comics—Even On-Line!

There are a total of 324 people who joined the Comic Gurus group on a social media site. Female group members outnumber males by 34. Determine how many males and females joined the Comic Gurus group.

1. Write an equation in standard form that represents the total number of people who joined the Comic Gurus group. Use x to represent the female members, and use y to represent the male members.

   $x + y = 324$

2. Write an equation in standard form to represent the number of female members in relationship to the number of male members.

   $x = y + 34 \rightarrow x - y = 34$

3. How are these equations the same? How are the equations different?

   Both equations have x and y on the same side. Both equations have an x coeficient of 1. A difference is that one of the y's is positive and the other is negative.

4. Complete parts (a) through (e) to write and solve a linear system of equations for this problem situation.

   a. Write a linear system for this problem situation.

   $$\begin{cases} x + y = 324 \\ x - y = 34 \end{cases}$$

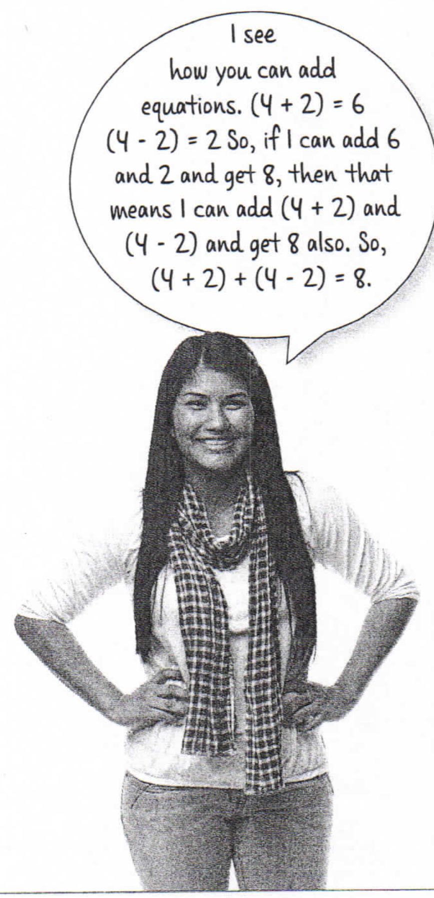
   b. Add the two equations together.

   $$\begin{array}{r} x + y = 324 \\ + \quad x - y = 34 \\ \hline 2x = 358 \end{array}$$

   c. Solve the resulting equation.

   $$\frac{2x}{2} = \frac{358}{2}$$

   $$\boxed{x = 179}$$



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d. Substitute the *x*-value that you obtained in part (c) into one of the original equations and solve to determine the value of *y*.

$x - y = 34$
$(179) - y = 34$
$-179 \quad -179$
$-y = -145$

$\frac{-y}{-1} = \frac{-145}{-1}$
$y = 145$

e. What is the solution of the linear system? Check your solution algebraically.

(179, 145)

$x + y = 324$
$(179) + (145) = 324$
$324 = 324$ ✓

$x - y = 34$
$(179) - (145) = 34$
$34 = 34$ ✓

5. Interpret the solution of the linear system in this problem situation.

The Comic Gurus group has 179 female members and 145 male members.

6. What effect did adding the equations together have?

Adding the equations together eliminated one of the variables entirely.

7. Describe how the coefficients of *y* in the original system are related.

The coefficients of y in the original system were opposites

* This is the key for this method to work

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PROBLEM 2 Let It Snow . . . For Winter Get-Aways

Let It Snow Resort offers two winter specials: the Get-Away Special and the Extended Stay Special. Let It Snow claims that the Extended Stay Special is the better deal. The Get-Away Special offers two nights of lodging and four meals for $270. The Extended Stay Special offers three nights of lodging and eight meals for $435. Determine if the Extended Stay Special is the better deal.

1. Write an equation in standard form that represents the Get-Away Special. Let *n* represent the cost for one night of lodging at the resort, and let *m* represent the cost for each meal. $2n + 4m = 270$

2. Write an equation in standard form that represents the Extended Stay Special. Use the same variables you used in Question 1.

   $3n + 8m = 435$

3. How are these equations the same? How are these equations different?

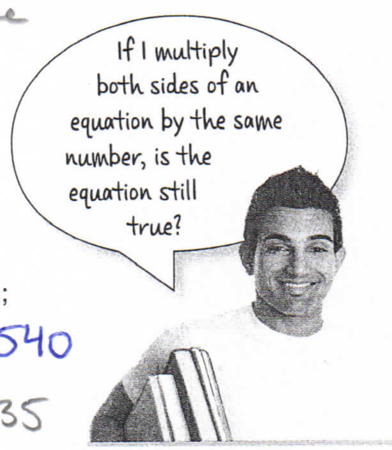
Both in standard form, all coefficients are positive, but none of the coefficients are the same, nor are they opposites.

4. Complete parts (a) through (h) to write and solve the system comparing the two winter specials.

   a. Multiply each side of the equation that represents the Get-Away Special by −2. Simplify the equation; maintain standard form.

   $-2 \cdot (2n + 4m = 270) \rightarrow -4n - 8m = -540$

   $3n + 8m = 435 \qquad 3n + 8m = 435$



   b. Write a linear system of equations using the transformed equation you wrote that represents the Get-Away Special and the equation that represents the Extended Stay Special.

   $-4n - 8m = -540$
   $3n + 8m = 435$

   c. How do the coefficients of the equations in your linear system of equations compare?

The coefficients of m are opposites now!

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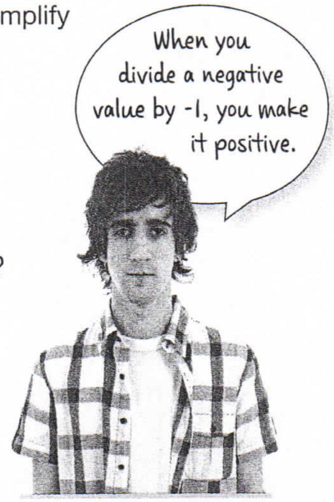
d. Add the equations in your linear system together. Then simplify the result. What does the result represent?

$+\ \ -4n - 8m = -540$
$3n + 8m = 435$

$-n = -105$
$\frac{-n}{-1} = \frac{-105}{-1}$

$n = 105$ Staying one night costs \$105.



e. How will you determine the *m*-value of the linear system?

I will plug in the value of n into one of the original equations.

f. Determine the value of *m* for the linear system.

$2n + 4m = 270$
$2(105) + 4m = 270$

$210 + 4m = 270$
$-210 \quad -210$
$4m = 60$
$\frac{4m}{4} = \frac{60}{4}$

$m = 15$

g. What is the solution of the linear system? Interpret the solution of the linear system in the problem situation.

(105, 15) It costs \$105 per night and \$15 per meal at Let it Snow Resort.

h. Check your solution algebraically.

$2n + 4m = 270$
$2(105) + 4(15) = 270$
$210 + 60 = 270$
$270 = 270$ ✓

$3n + 8m = 435$
$3(105) + 8(15) = 435$
$315 + 120 = 435$
$435 = 435$ ✓

5. Is the Extended Stay Special the better deal? Explain why or why not.

The Extended Stay Special is not the better option. Both specials cost \$105 per night and \$15 per meal, therefore they are the same, the extended stay is just one day longer and includes more meals.

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PROBLEM 3 Linear Combinations

The algebraic method you used to solve the linear systems in Problems 1 and 2 is called the *linear combinations method*. The **linear combinations method** is a process used to solve a system of equations by adding two equations together, resulting in an equation with one variable. You can then determine the value of that variable and use it to determine the value of the other variable.

In many cases, one or both of the equations in the system must be multiplied by a constant so that when the equations are added together, the result is an equation in one variable. This means that the coefficients of either the term containing x or y must be opposites.

Let's consider a system where neither of the x- or y-terms are opposites.

$$\begin{cases} 4x + 2y = 3 \\ 5x + 3y = 4 \end{cases}$$

Multiply each equation by a constant that results in opposite coefficients for one of the variables. →

$$3(4x + 2y) = 3(3)$$
$$-2(5x + 3y) = -2(4)$$

$$12x + 6y = 9$$
$$-10x - 6y = -8$$

Now that the y-values are opposites, you can solve this linear system.

1. Solve the new linear system shown in the worked example.

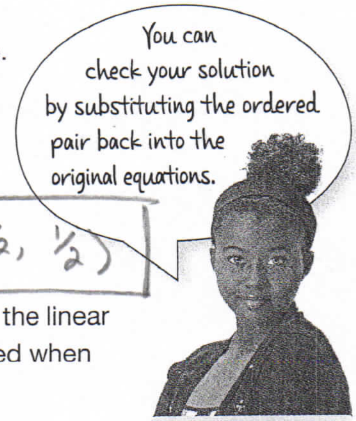
$12x + 6y = 9$
$+ \; -10x - 6y = -8$
$2x = 1$, $\frac{2x}{2} = \frac{1}{2}$

$x = \frac{1}{2}$

$12x + 6y = 9$
$12(\frac{1}{2}) + 6y = 9$
$6 + 6y = 9$

$6 + 6y = 9$
$-6 \quad -6$
$6y = 3$, $\frac{6y}{6} = \frac{3}{6}$
$y = \frac{1}{2}$

$(\frac{1}{2}, \frac{1}{2})$



2. Describe the first step needed to solve each system using the linear combination method. Identify the variable that will be solved when you add the equations.

   a. $5x + 2y = 10$ and $3x + 2y = 6$

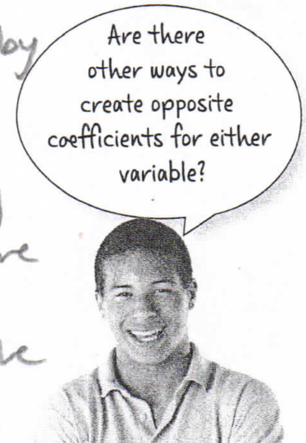
   I would multiply the second equation by −1, then would solve for x once the ys were eliminated.

   b. $x + 3y = 15$ and $5x + 2y = 7$

   I would multiply the first equation by −5, then solve for y once the xs were elimanated.

   c. $4x + 3y = 12$ and $3x + 2y = 4$

   I would multiply the first by 2 and the second by 3 so that the ys would be opposites, then I would be solving for x.



3. Solve each system using linear combinations.

a. $\begin{cases} 2x + y = 8 \\ 3x - y = 7 \end{cases}$

$5x = 15$

$\frac{5x}{5} = \frac{15}{5}$

$x = 3$

$2x + y = 8$

$2(3) + y = 8$

$6 + y = 8$

$-6 \quad -6$

$y = 2$

$(3, 2)$

b. $\begin{cases} 4x + 3y = 24 \\ -3(3x + y = -2) \end{cases}$

$4x + 3y = 24$

$-9x - 3y = 6$

$-5x = 30$

$\frac{-5x}{-5} = \frac{30}{-5}$

$x = -6$

$3x + y = -2$

$3(-6) + y = -2$

$-18 + y = -2$

$+18 \quad +18$

$y = 16$

$(-6, 16)$

c. $\begin{cases} -2(3x + 5y = 17) \\ 3(2x + 3y = 11) \end{cases}$

$-6x - 10y = -34$

$6x + 9y = 33$

$-y = -1$

$\frac{-y}{-1} = \frac{-1}{-1}$

$y = 1$

$2x + 3y = 11$

$2x + 3(1) = 11$

$2x + 3 = 11$

$-3 \quad -3$

$2x = 8$

$\frac{2x}{2} = \frac{8}{2}$

$x = 4$

$(4, 1)$

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Be prepared to share your solutions and methods.