key

Lesson 5: The Zero Product Property

Classwork

Opening Exercise

Consider the equation $a \cdot b \cdot c \cdot d = 0$. What values of a, b, c, and d would make the equation true?

at least one of the variables have to be zero!

Zero Product Property

if $a \cdot b = 0$,
then $a=0$ or $b=0$ or $a=b=0$

Exercises 1–4

Find values of c and d that satisfy each of the following equations. (There may be more than one correct answer.)

1. $cd = 0$

$C=0$ or $d=0$

c	d
0	0
7	0
0	100
0	1

$4 \cdot \frac{1}{2} = 2$

2. $(c-5)d = 2$

c	d
7	1
6	2
9	½

3. $(c-5)d=0$

c	d
5	d
c	0

$c=5$ or $d=0$

4. $(c-5)(d+3)=0$

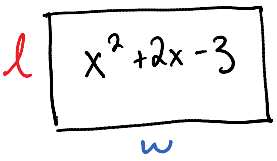
c	d
5	d
c	-3

$c=5$ or $d=-3$

Example

For each of the related questions below, use what you know about the zero product property to find the answers.

a. The area of a rectangle can be represented by the expression x^2+2x-3. Write each dimension of this rectangle as a binomial, and then write the area in terms of the product of the two binomials.

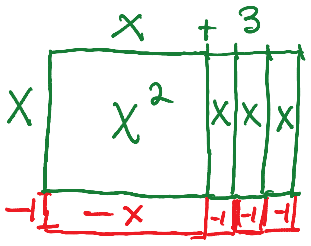


Area = $l \cdot w$

$x^2+2x-3=(x-1)(x+3)$

length = $x-1$
width = $x+3$

b. Can we draw and label a diagram that represents the rectangle's area?



$A=x^2+2x-3$

c. Suppose the rectangle's area is known to be 21 square units. Can you find the dimensions in terms of x?

Area = (l)(w)
21 = (l)(w)
21 = (x-1)(x+3)

$21 = (x-1)(x+3)$ → 3 · 7

d. Rewrite the equation so that it is equal to zero and solve.

Area = x^2+2x-3
21 = x^2+2x-3
−21 −21
$0 = x^2+2x-24$

set equal to zero

$0 = x^2+2x-24$
$0 = (x-4)(x+6)$
$x-4=0$ or $x+6=0$
$x=4$ or ~~$x=-6$~~ Not a "real" solution

e. What are the actual dimensions of the rectangle?

length = $x-1 \rightarrow (4)-1 = 3$
width = $x+3 \rightarrow (4)+3 = 7$

f. A smaller rectangle can fit inside the first rectangle, and it has an area that can be expressed by the equation x^2-4x-5. What are the dimensions of the smaller rectangle in terms of x?

$A = l \cdot w$
$x^2-4x-5 = (x-5)(x+1)$
l w

g. What value for x would make the smaller rectangle have an area of $\frac{1}{3}$ that of the larger?

Area = x^2-4x-5
$(\frac{1}{3})21 = x^2-4x-5$
$7 = x^2-4x-5$
−7 −7

set equal to zero

$0 = x^2-4x-12$
$0 = (x-6)(x+2)$
$x=6$ or ~~$x=-2$~~

Exercises 5–8

Solve. Show your work.

5. $x^2 - 11x + 19 = -5$

1. set equal to zero

$+5 \quad +5$

$x^2 - 11x + 24 = 0$

2. factor
3. solve

$a = 1$
$b = -11$
$c = 24$

a·c = 24

$-8x$ and $-3x$ (x and -11)

$x^2 - 8x - 3x + 24 = 0$

	x	-3
x	x^2	$-3x$
-8	$-8x$	24

$(x-3)(x-8) = 0$

now solve for x

$x = 3$ or $x = 8$

6. $7x^2 + x = 0$

$x(7x+1) = 0$

$x = 0$ or $7x + 1 = 0$

$-1 \quad -1$

$\frac{7x}{7} = \frac{-1}{7}$

$x = 0$ or $x = -\frac{1}{7}$

7. $7r^2 - 14r = -7$

$r = 1$

8. $2d^2 + 5d - 12 = 0$

$(2d-3)(d+4) = 0$

$d = \frac{3}{2}$ or $d = -4$

Lesson Summary

Zero Product Property

If $ab = 0$, then $a = 0$ or $b = 0$ or $a = b = 0$.

When solving for the variable in a quadratic equation, rewrite the equation as a factored quadratic set equal to zero. Using the zero product property, you know that if one factor is equal to zero, then the product of all factors is equal to zero.

Going one step further, when you have set each binomial factor equal to zero and have solved for the variable, all of the possible solutions for the equation have been found. Given the context, some solutions may not be viable, so be sure to determine if each possible solution is appropriate for the problem.

Problem Set

Solve the following equations.

1. $x^2 + 15x + 40 = 4$
2. $7x^2 + 2x = 0$
3. $b^2 + 5b - 35 = 3b$
4. $6r^2 - 12r = -6$

1) $x^2 + 15x + 36 = 0$

$(x+12)(x+3) = 0$

$x+12=0$ or $x+3=0$

$x = -12$ or $x = -3$

2) $7x^2 + 7x = 0$

$7x(x+1) = 0$

$7x = 0$ or $x+1 = 0$

$x = 0$ or $x = -1$

3) $b^2 + 5b - 35 = 3b$

$-3b \quad -3b$

$b^2 + 2b - 35 = 0$

$(b+7)(b-5) = 0$

$b+7=0$ or $b-5=0$

$b = -7$ or $b = 5$

4) $6r^2 - 12r = -6$

$+6 \quad +6$

$6r^2 - 12r + 6 = 0$

$6(r^2 - 2r + 1) = 0$

$6(r-1)(r-1) = 0$

$r-1=0$ or $r-1=0$

$r = 1$ or $r = 1$